EXERCISE 1

CHARGE STABILITY DIAGRAM OF A SINGLE QD

1) $Q_G = C_G (V_G - V_1)$ (a)

$Q_1 = C_1 V_1$ (b)

$Q_2 = C_2 V_2$ (c)

$Q_1 - Q_2 - Q_G = - N|e|$ (d)

5th eq. from the circuit: $V_1 + V_2 = V_D$ (e)

$$E_{el} = \frac{1}{2}\left(Q_G V_G - N|e| V_1 + Q_2 V_D\right) \quad (f)$$

a), b), c) into d):

$$C_1 V_1 - C_2 \underbrace{V_2}_{\to\, V_D - V_1 \ (e)} - C_G (V_G - V_1) = - N|e|$$

$$(C_1 + C_2 + C_G) V_1 = C_2 V_D + C_G V_G - N|e|$$

$$\to V_1 = \frac{C_G V_G + C_2 V_D - N|e|}{C_1 + C_2 + C_G} \quad (g)$$

$$\to V_2 = V_D - V_1 = \frac{C_1 V_D + \cancel{C_2 V_D} + C_G V_D - C_G V_G - \cancel{C_2 V_D} + N|e|}{C_1 + C_2 + C_G} =$$

$$= \frac{(C_1 + C_G) V_D - C_G V_G + N|e|}{C_1 + C_2 + C_G} \quad (h)$$

(g), (h), (a), (c) into (f)

$$E_{el} = \frac{1}{2}\left(C_G\left(V_G - \frac{C_G V_G + C_2 V_S - N|e|}{C_1 + C_2 + C_G}\right)V_G - N|e|\,\frac{C_G V_G + C_2 V_S - N|e|}{C_1 + C_2 + C_G} + \right.$$

$$\left. + C_2\,\frac{(C_1 + C_G)V_S - C_G V_G + N|e|}{C_1 + C_2 + C_G}\,V_D\right) =$$

$$= \frac{1}{2}\left(\frac{C_1 C_G V_G^2 + C_2 C_G V_G^2 + \cancel{C_G^2 V_G^2} - \cancel{C_G^2 V_G^2} - C_G C_2 V_D V_G + \cancel{N|e| C_G V_G}}{C_1 + C_2 + C_G} + \right.$$

$$\left. + \frac{-N|e|\cancel{C_G V_G} - N|e|\cancel{C_2 V_D} + N^2 e^2 + C_1 C_2 V_D^2 + C_2 C_G V_D^2 - C_G C_2 V_G V_D + \cancel{N|e| C_2 V_D}}{C_1 + C_2 + C_G}\right) =$$

$$= \frac{C_1 C_G V_G^2 + C_1 C_2 V_D^2 + C_2 C_G (V_G - V_D)^2 + N^2 e^2}{2(C_1 + C_2 + C_G)}$$

$$E_{el}(N) = \frac{N^2 e^2}{2(C_1 + C_2 + C_G)}$$

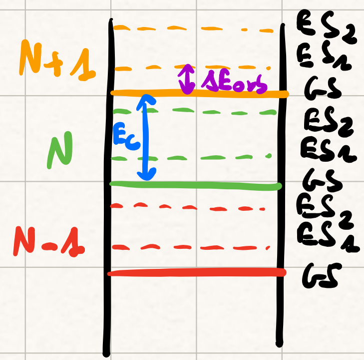
If I want to add an extra electron:

$$\Delta E_{el}(N) = E_{el}(N+1) - E_{el}(N) = \frac{(N+1)^2 e^2 - N^2 e^2}{2(C_1 + C_2 + C_G)} =$$

$$= \frac{\cancel{N^2 e^2} + 2Ne^2 + e^2 - \cancel{N^2 e^2}}{2(C_1 + C_2 + C_G)} = \frac{e^2}{C_1 + C_2 + C_G}\left(N + \frac{1}{2}\right) = \Delta E_c(N)$$

This quantity is called "charging energy", because it tells how much energy we have to spend to add an extra electron to the dot if we have already N electrons inside.

This value corresponds to the "addition energy", in case we are neglecting excited states which arise from quantum confinement:



Notice that in principle there are infinite excited states

$$E_{add} = E_c + \Delta E_{orb}$$

$$\Delta E_{orb} \sim \frac{\hbar^2}{m^* L^2}$$

(particle in a box like)

Since $E_c > \Delta E_{orb}$ usually, in all these computations coming from this simple capacitance model we are neglecting effects of quantum confinement ($\Delta E_{orb} = 0$, meaning that discretization rules are dictated just by coulomb repulsion, since it usually dominates).

2) $$\Delta E_{gen} = -\delta q_G V_G - \delta q_2 V_D = -C_G \delta(\overbrace{\cancel{V_G}}^{0\ \text{(not depending on } N)} - V_1) V_G - C_2 \delta V_2 V_D$$

$$= + C_G \delta V_1 V_G - C_2 \delta V_2 V_D = C_G V_G \frac{-(N+1)|e| + N|e|}{C_1+C_2+C_G} +$$

$$- C_2 V_D \frac{(N+1)|e| - N|e|}{C_1+C_2+C_G} = - \frac{|e|}{C_1+C_2+C_G}\left(C_G V_G + C_2 V_D\right)$$

$$\longrightarrow \Delta E_{tot} = \frac{1}{C_1+C_2+C_G}\left(N e^2 + \frac{1}{2} e^2 - |e|\left(C_G V_G + C_2 V_D\right)\right) =$$

$$= \frac{|e|}{C_1+C_2+C_G}\left(\left(N+\frac{1}{2}\right)|e| - C_G V_G - C_2 V_D\right)$$

3) To inject one electron from the source to the dot:

$$\Delta E_{tot} < 0 \longrightarrow \left(N+\frac{1}{2}\right)|e| - C_G V_G - C_2 V_D < 0$$

$$-C_2 V_D < -\left(N+\frac{1}{2}\right)|e| + C_G V_G$$

$$\longrightarrow V_D > -\frac{C_G}{C_2} V_G + \left(N+\frac{1}{2}\right)\frac{|e|}{C_2} \qquad (i)$$

4) Inject an electron from dot to the drain:

$$\Delta E_{tot}\,(N \to N-1) < 0$$

$$\Delta E_c\,(N \to N-1) = -\left[\frac{N^2 e^2}{2(C_1+C_2+C_G)} - \frac{(N-1)^2 e^2}{2(C_1+C_2+C_G)}\right] = -\frac{e^2}{C_1+C_2+C_G}\left(N-\frac{1}{2}\right)$$

$$\Delta E_{gen}\,(N \to N-1) = -\delta q_G V_G + \delta Q_D V_D = -\delta q_G V_G + \delta(-q_1 + q_G) V_D =$$

$$= -\underline{\delta q_G V_G} - \underline{\delta q_1 V_D} + \underline{\delta q_G V_D} \overset{(\delta V_G = 0)}{=} + \underline{C_G \,\delta V_1 V_G} - \underline{C_1 \,\delta V_1 V_D} + $$

$$- \underline{C_G \,\delta V_1 V_D} = \delta V_1 \left(C_G V_G - C_1 V_D - C_G V_D\right) =$$

$$= \frac{N|e| - (N-1)|e|}{C_1+C_2+C_G}\left(C_G V_G - (C_1 + C_G) V_D\right) = \frac{|e|}{C_1+C_2+C_G}\left(C_G V_G - (C_1+C_G)V_D\right)$$

$$\Delta E_{tot} = \frac{|e|}{C_1+C_2+C_G}\left(-\left(N-\frac{1}{2}\right)|e| + C_G V_G - (C_1+C_G) V_D\right)$$

5) $\Delta E_{tot} < 0 \longrightarrow -(C_1+C_G) V_D < \left(N-\frac{1}{2}\right)|e| - C_G V_G$

$$\longrightarrow V_D > -\left(N-\frac{1}{2}\right)\frac{|e|}{C_1+C_G} + \frac{C_G}{C_1+C_G} V_G \qquad (ii)$$

6-7) (i), (1) are eq. of a line in the $V_S - V_G$ plane (one line for each N value)

However, to get the full picture, we are missing one point: what if $V_D < 0$?

In fact, if $V_D < 0$, it is more energetically favorable to inject one electron from the drain to the dot and then from the dot to the source (the opposite with respect to what we got so far).

If we do the computation, we get two new inequalities:

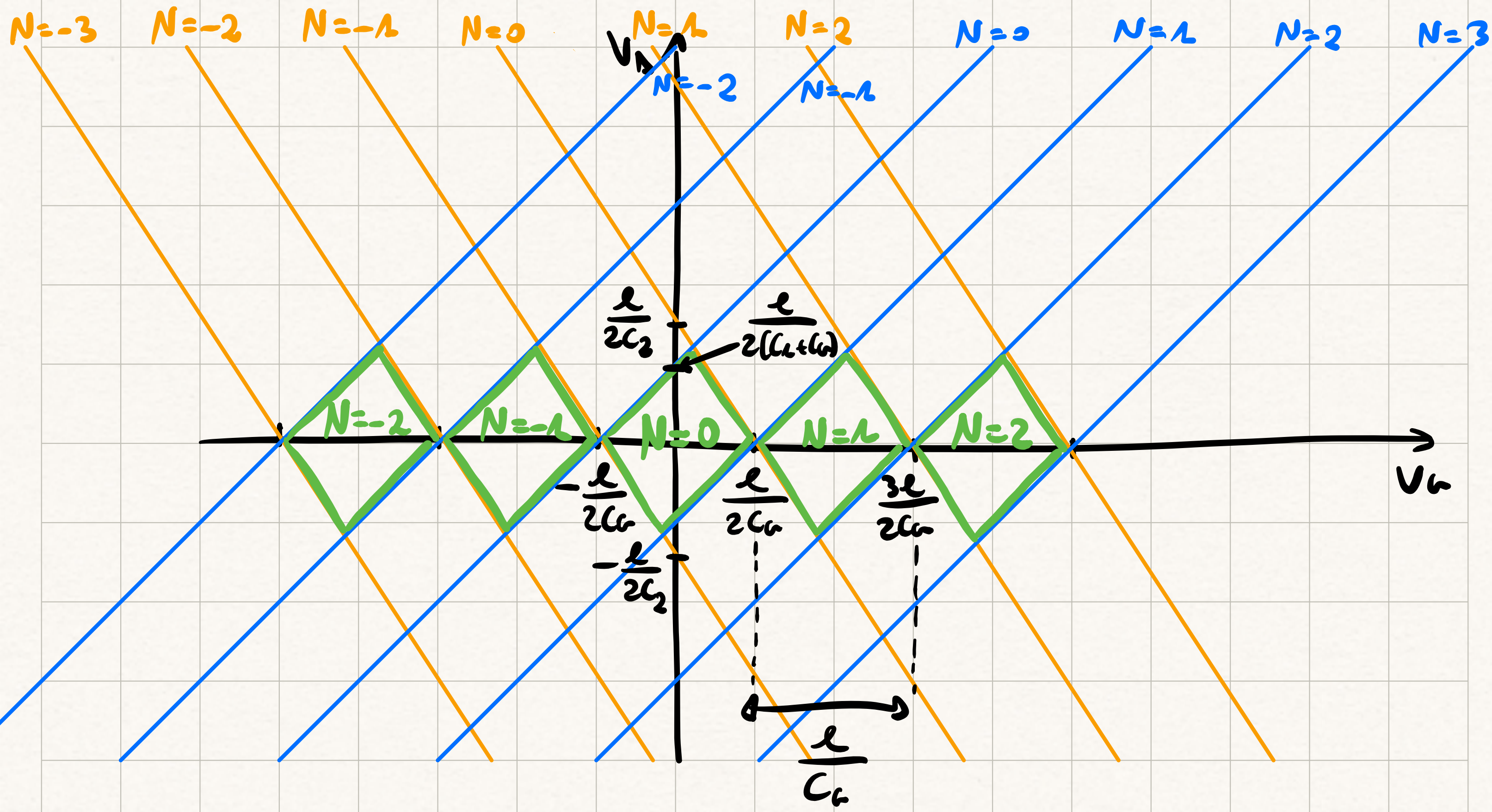
- D to dot $\Leftrightarrow$ $V_D < \frac{C_G}{C_1 + C_G} V_G - \left(N + \frac{1}{2}\right) \frac{|e|}{C_1 + C_G}$

- dot to S $\Leftrightarrow$ $V_S < -\frac{C_G}{C_2} V_G + \left(N - \frac{1}{2}\right) \frac{e}{C_2}$

So, in the end, we get 4 inequalities which have to be satisfied to get a current from S to D (for $V_D > 0$) or from D to S (for $V_D < 0$)

$$\left.\begin{aligned} &\cdot\; V_S > -\frac{C_G}{C_2} V_G + \left(N+\frac{1}{2}\right)\frac{|e|}{C_2} \quad (V_D > 0) \\ &\cdot\; V_S < -\frac{C_G}{C_2} V_G + \left(N-\frac{1}{2}\right)\frac{e}{C_2} \quad (V_D < 0) \end{aligned}\right] \quad \begin{matrix}\text{source}\\ \Updownarrow \\ \text{dot}\end{matrix}$$

$$\left.\begin{aligned} &\cdot\; V_D > \frac{C_G}{C_1 + C_G} V_G - \left(N-\frac{1}{2}\right)\frac{|e|}{C_1 + C_G} \quad (V_D > 0) \\ &\cdot\; V_D < \frac{C_G}{C_1 + C_G} V_G - \left(N+\frac{1}{2}\right)\frac{|e|}{C_1 + C_G} \quad (V_S < 0) \end{aligned}\right] \quad \begin{matrix}\text{drain}\\ \Updownarrow \\ \text{dot}\end{matrix}$$

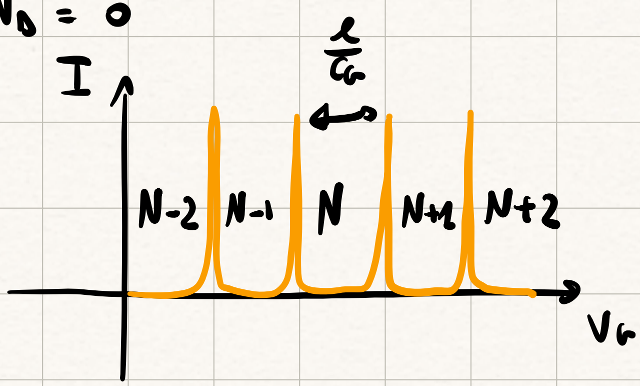


So, if we plot these inequalities for each value of N, we get finally the "charge stability diagram" of a single quantum dot. The green regions, called "Coulomb diamonds" because of their shape, are regions where electron transport is completely forbidden by coulomb repulsion. Outside the diamonds, a current can flow from S to D and viceverse (according to the sign of V_D).

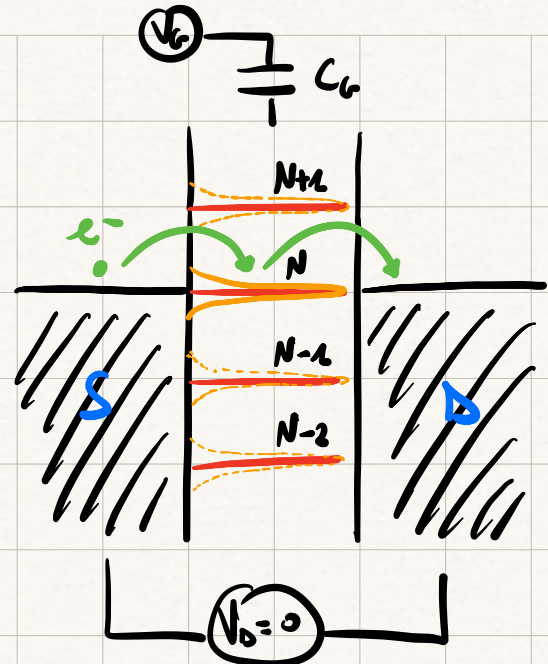
In conclusion, we have to provide energy to electrons in order to overcome coulomb repulsion. We can do it through V_G or/and V_D

• <u>Effect of V_G</u> :

- $V_D = 0$



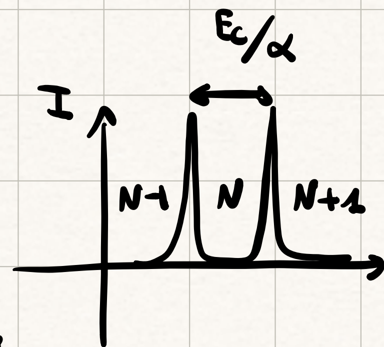
Coulomb oscillations



V_G is able to move up and down the chemical potential of the dot. When this potential is aligned to the chemical potentials of the reservoirs, electron tunneling is possible.

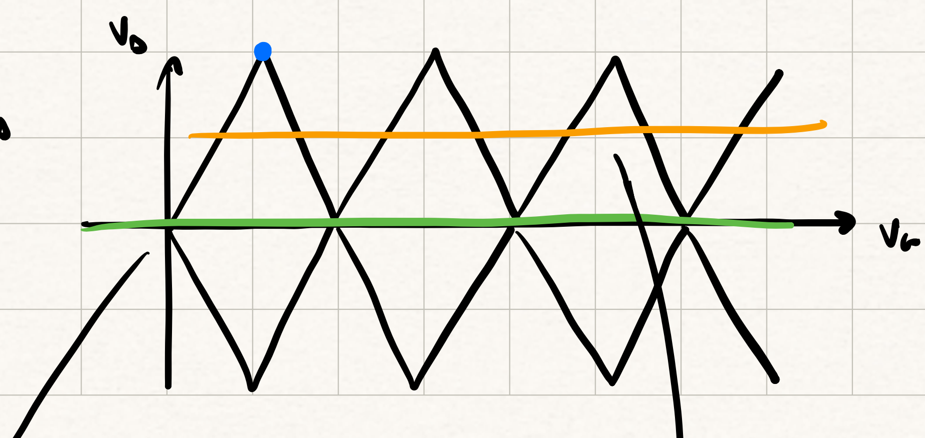
How to convert gate voltage V_G directly into energy provided to the electrons? Through the lever arm α:

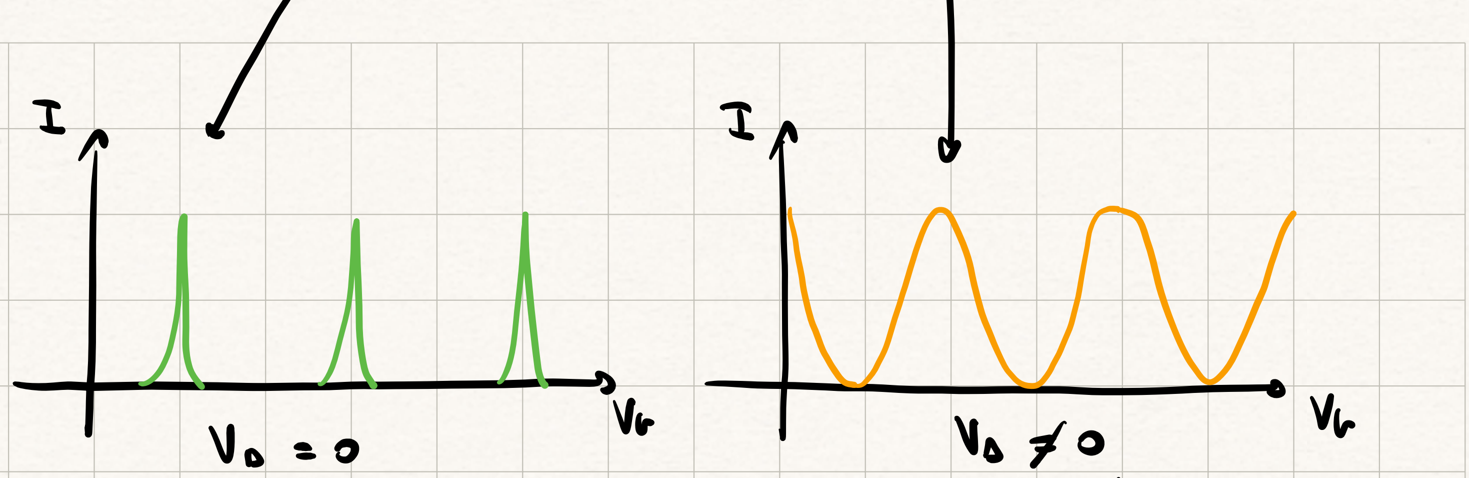
$$\alpha = \frac{C_G}{C_1+C_2+C_G}|e| \quad \left(\text{in } \frac{meV}{mV}\right)$$

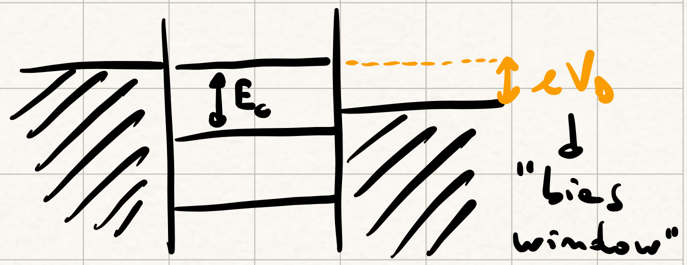


Through α : $E_C = \alpha \frac{e}{C_G}$ → V_G periodicity

- Effect of V_D



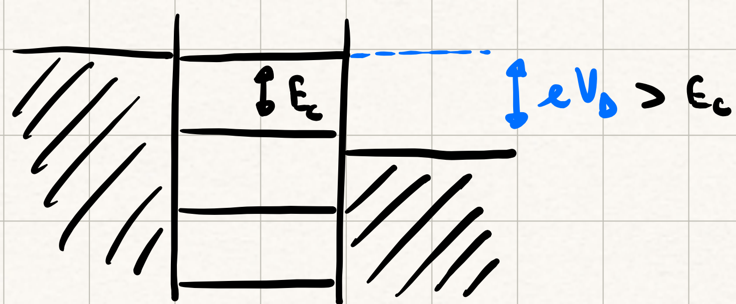




Blue point: no confinement any more

$\rightarrow$ $\underline{E_c = eV_D}$

The bias window is so large that there is always an energy level in the dot within the window, independently on V_G



How much is this value?

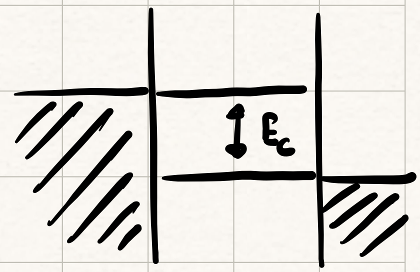
From eq. (i) and (1):

$$\begin{cases} V_D = -\dfrac{C_G}{C_2} V_G + \left(N+\dfrac{1}{2}\right)\dfrac{|e|}{C_2} \longrightarrow V_G = -\dfrac{C_2}{C_G} V_D + \left(N+\dfrac{1}{2}\right)\dfrac{|e|}{C_G} \\ V_D = -\left(N-\dfrac{1}{2}\right)\dfrac{|e|}{C_1+C_G} + \dfrac{C_G}{C_1+C_G} V_G \longrightarrow V_G = \dfrac{C_1+C_G}{C_G} V_D + \left(N-\dfrac{1}{2}\right)\dfrac{|e|}{C_G} \end{cases}$$

$$\rightarrow -\frac{C_2}{C_G}V_D + (\cancel{N}+\tfrac{1}{2})\frac{|e|}{C_G} = \frac{C_1+C_G}{C_G}V_D + (\cancel{N}-\tfrac{1}{2})\frac{|e|}{C_G}$$

$$(C_1+C_2+C_G)V_D = |e| \longrightarrow V_D = \frac{|e|}{C_1+C_2+C_G}$$

$$\rightarrow E_c = eV_D = \frac{e^2}{C_1+C_2+C_G}$$

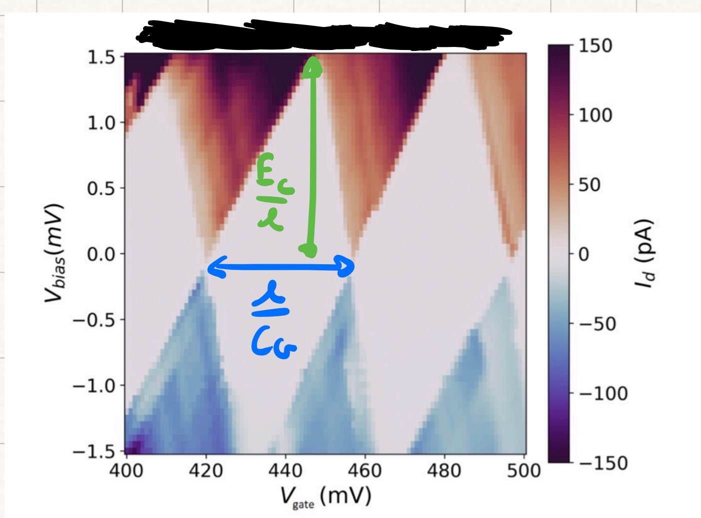


Notice that this value of E_c is different from the expression we found previously in the exercise, which was N-dependent $\left(\Delta E_c(N) = \frac{e^2}{C_1+C_2+C_G}(N+\frac{1}{2})\right)$

This is because it is more convenient to re-define an expression which is N-independent, in order to think in terms of energy differences and not absolute values defined starting from an arbitrary reference (ex. N=0)

$$E_c = \Delta E_c(N+1) - \Delta E_c(N) = \frac{e^2}{C_1+C_2+C_G} = \frac{e^2}{C_{dot}}$$

8)



$E_c \simeq 1{,}5$ meV

$\Delta V_G = \frac{e}{C_G} \simeq 36$ mV

$$\alpha = e\frac{C_G}{C_{dot}} = \frac{e^2/\Delta V_G}{e^2/E_c} = \frac{E_c}{\Delta V_G} \quad (\rightarrow \Delta V_G = E_c/\alpha)$$

$\rightarrow \alpha \simeq 0.04 \frac{meV}{mV} \rightarrow$ if we apply $V_G = 1$ mV, we shift the chemical potential in the dot of 40 μV

Notice that now that we know E_c, we can also compute the total capacitance of the dot:

$$E_c = \frac{e^2}{C_{dot}} \rightarrow C_{dot} = \frac{e^2}{E_c}$$

And by approx. the dot as a conductive disk, we can estimate the dot radius:

$$C_{dot} = 8 \varepsilon_0 \varepsilon_r R_{dot}$$

Notice: in the picture you can also see other transition lines outside the diamond, which lead to an increase in current when matching this transition by V_G and V_S $\rightarrow$ excited states due to quantum confinement, as anticipated at the beginning $\rightarrow \Delta E_{orb}$ can be extracted from the graph as well

Max T ?

$$E_c > k_B T \rightarrow \boxed{T < \frac{E_c}{k_B}} = \frac{1{,}5 \text{ meV}}{8{,}6 \cdot 10^{-5} \text{ eV/K}} = \boxed{17 \text{ K}}$$